For the trip to Alpha Centauri,

a) how long does the outward trip take to the Earth twin, Jeff?

t=d/v = 4.5 light years/0.99c = 4.5/0.99=4.5455 years

b) how far is the outward trip to the travelling twin, Marco?

Marco sees Earth and Alpha Centauri move at 0.99c, so the length is contracted by gamma.

$x'=\gamma(x-vt)$ alternately, $L=L_o/\gamma$

$\gamma=1/\text{root}(1-v^2/c^2)$

$= 1/\text{root}(1-0.99^2 \cancel{(c^2/c^2)})$

0.99x0.99=0.9801

1-0.9801=0.0199

Sqrt(0.0199)=0.141067359796659

1/0.14106735979665=7.0888120500838

$\gamma=7.089$ (no units)

$L=4.5\text{lightyears}/\gamma$ = 4.5/7.088812050083=

0.634803119085

the distance is only $\boxed{\text{0.63light years}}$ in Marco's frame

c) how long does the outward trip take for Marco?

0.63lightyears/0.99c = 0.634803119085/0.99=

0.641215271803 years = $\boxed{\text{0.64years}}$

d) What year is in on Earth when Marco turns around? (tricky)

to Jeff it is 4.5455 years later

$t'=\gamma(t-vx/c^2)$

$t'=7.09(0.64y+0.99cx4.5cy/c^2)$

t'= 7.09x(0.64+(0.99x0.64))=9.0298 years

t'= 7.09x(0.64-(0.99x0.64))=0.0454 years

which time is valid?

both, they depend on the spacetime experienced.

Marco shifts from one spacetime to another when he turns around, so time on Earth shifts - Minkowski diagrams

a) How long does the return trip take in each frame of reference?
   4.5455 years to Jeff
   0.64years to Marco

round trip for Jeff - 4.5455x2=9.091 years
round trip for marco 0.641215271803x2= 1.282430543606 = 1.3 years

Jeff aged 9 years while Marco aged 1 year
https://www.youtube.com/watch?v=ewfRxjesNTs
simultanaety - events that happen at the same time in one frame, are at different times in another frame depending on the v and x.

evidence?

muons live 2.2 micro seconds, but when they are moving near the speed of light, they live longer.

addition of velocities:

$u'=(u-v)/(1-uv/c^2)$

eg. You are in a ship moving at 0.60c relative to the Earth. Another ship is moving in the other direction, at -0.60c relative to the Earth. What is the speed of each ship in the other's frame of reference?

$u'=(u-v)/(1-uv/c^2) = (0.60c - (-0.60c))/(1-(-0.6x0.6))$
$1.2c/(1+0.36)=0.8824\ c$ so they are moving at 0.88c in each other's frame of reference.

so this means nothing can accelerate to the c, no matter how much energy you put in.

Where does the energy go?

One way of thinking of it , is it goes into mass/energy $E=mc^2$ total energy = $\gamma m_o c^2$
rest mass energy $E_o = m_o c^2$
or the kinetic energy $E_k = E - E_o$
$E_k = mc^2 - m_o c^2 = (\gamma - 1) m_o c^2$

m_o is the rest mass - no relative motion

$p=\gamma m_o v$

http://ibdiploma.cambridge.org/user/login

p27 in Cambridge study guide
Q17, 19, 21, 24
p35 Q32, 37, 40
p61 Q4 tricky

eg. an electron, mass 9.11×10^{-31}kg is moving at 0.90c. What is the

a) rest mass energy, in Joules
b) kinetic energy
c) total energy
d) relativistic momentum